Multiplier and Velocity Analysis: A Marriage

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I

INTRODUCTION

More than ten years ago Professor Samuelson spoke of the unsuccessful attempts that had been made by numerous writers to reconcile multiplier analysis with velocity analysis, attempts which he judged to be either " trivial " or " founded upon error ".[1] It does not seem that during the interval much has happened to modify this view, which is still perhaps that of a majority of economists. Professor Machlup was anxious to defend himself against Samuelson's[2] accusation that he " was among those attempting a ' marriage ' between the velocity and the multiplier approaches " by explaining that his velocity discussion served " merely to arrive at an estimate of the time dimensions of the multiplier effects ".[3] And more recently, Professor Ackley[4] has complimented Mr. Goodwin[5] on having been " careful to recognise that 'velocity has no explanatory value ' but rather merely that ' multiplier analysis can make important use of the rich empirical evidence from monetary studies ' ". Professor Angell's rather lengthy discussion,[6] now more than twelve years old, of some of the connections between multiplier and velocity analysis, in terms which could hardly be called trivial, has passed largely unnoticed.

In this article we shall be concerned purely with certain formal aspects of multiplier analysis, of which we shall represent velocity analysis as one variety. We shall try to show that velocity can be explained in terms of a multiplier sequence, found by a process analogous to that by which the investment multiplier sequence is found, but based on a hypothesis about the nature of the hoarding function which, although probably no less reasonable than that made about the consumption function in the investment multiplier case, gives us the result that velocity tends towards stability unless the function shifts. This is a result which corresponds to what many " velocity

[1] Paul A. Samuelson, " Fiscal Policy and Income Determination ", *Quarterly Journal of Economics*, 1941-42, p. 602.

[2] *Ibid.*, p. 603.

[3] Fritz Machlup, " Period Analysis and Multiplier Theory ", *Readings in Business Cycle Theory* (Philadelphia, 1944), p. 220n.

[4] Gardner Ackley, " The Multiplier Time Period : Money, Inventories and Flexibility ", *American Economic Review*, 1951, p. 350.

[5] Richard M. Goodwin, " The Multiplier ", *The New Economics* (ed. Seymour Harris, New York, 1947), pp. 488-499.

[6] James W. Angell, *Investment and Business Cycles* (New York and London, 1941), especially Chapters IX, X and XI.

theorists " believed but which some " multiplier theorists " claim to have disproved.

Since we shall be concerned exclusively with multiplier *mechanics* we shall formulate the whole of the argument in terms of money flows. We shall assume that no price changes occur, and no relevant changes in income or assets distribution, since such changes are likely to induce shifts in the relevant (aggregate) functions and therefore to introduce a factor which is not germane to our main argument and which we do not propose to treat.

We shall be concerned only with expansionary processes; and we shall treat our problem in terms of a closed system.

II

Alternative Multiplier Equations

Starting from the relationship between money income, and consumption and investment expenditures expressed in the familiar identity

$$Y = C + I$$

where Y is the money income of a given period, C is the consumption of that period and I the investment, we may establish various alternative " multiplier " equations based on different divisions of the expenditures. For the present we shall consider three such equations.

The first, which has by the usage of two decades come to be called *the* multiplier equation, obtains the figure for income by taking investment as the multiplicand, and using as the multiplier the expression $\frac{1}{1-\alpha}$, where α represents the ratio of consumption to income, or the (average) " propensity to consume ", $\frac{C}{Y}$, and $1 - \alpha$ the (average) " propensity to save ". This we shall call, following Keynes' original terminology, the " investment multiplier ".

A second formula, which has only rarely been used in the literature,[1] takes consumption as the multiplicand; and the multiplier expression is then $\frac{1}{1-\beta}$, where β represents the (average) " propensity to invest ", or, that is, $\frac{I}{Y}$, and $1 - \beta$ what Lange[2] called the " reluctance to invest ". This may, by analogy with the first, be called the " consumption multiplier ".

The third expression, which has crept into the literature sometimes overtly and sometimes only hidden between the lines, identifies the multiplicand with the amount of new money, M_1, created in the period,

[1] Cf., however, Oscar Lange. " The Theory of the Multiplier ", *Econometrica*, 1943, pp. 227ff.

[2] *Ibid.*

independently of whether that money is initially spent for investment or consumption purposes. The multiplier is then $\frac{1}{1-\gamma}$, where γ stands for the (average) " propensity to spend ", again whether for consumption or investment purposes, and $1-\gamma$ for the " propensity to hoard ". In this case I and C in the income equation are each divided into two components, which we may denote by the subscripts a and b respectively, so that if the multiplicand is written $M_1 = I_a + C_a$, $\frac{1}{1-\gamma}$ or what we may call the " new money multiplier " may be written equal to $\frac{I_b + C_b}{Y}$, the identity between income and total expenditures thus being again preserved. The multiplier is here a form of income velocity, different however from that which corresponds to the accepted meaning of the term. We may write this third multiplier equation

$$Y = M_1 V_1$$

where $V_1 = \frac{1}{1-\gamma}$.[1]

Our third expression evidently bears a close relationship to a fourth multiplier equation, which has a longer history than any of the first three. This is the familiar equation of " quantity theory ", $Y = MV$, where M represents the total quantity of money outstanding, and V the " money multiplier ", or the income velocity of circulation in the usual sense. In this fourth equation the multiplicand, M, includes all past creations of money, instead of, as in the third, only the money newly created in the period. The multipliers are correspondingly different: for since $Y = MV = M_1V_1$, it follows that $V = \frac{M_1}{M} V_1$. We may also write $MV = M_1V_1 + M_2O$ implying a division of the total stock of money into two parts, new and old, which is tantamount to regarding the velocity of the old money (M_2) as zero, or, that is, to treating the old money as though it had all gone into hoards. It may be noted that this procedure of separating out the new from the old money implied by the third multiplier equation has its exact counterpart in the investment multiplier equation: for there too investments which took place prior to the period concerned are treated as being irrelevant to the determination of the income of that period.

We thus have three multiplier equations, all satisfying the identity $Y = C + I$, as follows:—

(1) $Y = \frac{1}{1-\alpha} I$, where $I > O$ and $O < \alpha < 1$;

[1] γ in this expression has been identified by some writers with the sum of the propensities to consume and invest, i.e. with $\alpha+\beta$. We shall comment on certain inconsistencies of this treatment in a later section (see p. 42 below).

(2) $Y = \frac{1}{1 - \beta} C$, where $C > O$ and $O < \beta < 1$;

(3) $Y = \frac{1}{1 - \gamma} M_1$, where $M_1 > O$ and $O < \gamma < 1$;

and we have

(4) $Y = MV$.

All of the first three equations as thus represented are simple rearrangements of the equation from which we started out: like the fourth they are mere definitional or statistical relationships without any causal significance. The multipliers are " static " multipliers.

The principal claim of multiplier analysis is, however, to have provided an explanation of income generation and propagation in terms of a causal process, or dynamic multiplier series, depicting the successive " rounds " of additional income generated by a unit of additional expenditure of the type defined by the relevant multiplicand; a series which, provided we can assume constancy of the marginal propensity[1] to consume, invest or spend, takes the form of the simplest kind of geometric series, and which, under certain circumstances (namely that the marginal propensity is less than unity and that the series is drawn out to an infinite number of terms), sums to a limiting value identical with the corresponding " static " multiplier expression. It is this dynamic explanation which " velocity theorists " are charged by " multiplier theorists " with having failed to give in respect of the formula $Y = MV$.

In each case the working of the multiplier series through time presupposes a lag, or " multiplier period ", between one income " round " and the next. And the customary simplifying procedure has been to assume that the lag is a single one (rather than distributed) and to relate the consumption (or investment, or spending) of any unit period to the income received in the previous period (rather than to a series of past incomes or even expected future incomes). The period has been fairly widely identified with the inverse of what Professor Angell has called the " average circular velocity (per annum) of active money ",[2] resting on the supposition that there is an average " normal " period, determined largely by technical and institutional factors, which it takes for money to complete a circuit from one income recipient to another—a period which, unless conditions are very unstable, changes only very slowly in the course of time. A substantial part of the literature on the investment multiplier has, it is true, been concerned with the

[1] In order to keep our notation as simple as possible we shall not introduce a different set of symbols to distinguish marginal from average propensities, etc.

[2] Angell, it should be noted, defines " additions to hoards " as " money which is withheld from any sort of spending on goods and services " (*op. cit.*, p. 132), so that " hoards " include funds which are active in a " transactions " sense (as for example in the financial circulation) but not active in income creation.

difficulty of defining this period rigorously.[1] All that we need remark here, however, is that similarity of the period concept is the one point of contact which has fairly generally been admitted between multiplier and velocity analysis, and that any difficulties of definition that may arise are common to both.

III

The Inference from Multiplier Theory of a Dwindling Velocity

Multiplier theorists have drawn certain conclusions from the results of applying to new money the multiplier $\frac{1}{1-\gamma}$, at which they arrived by direct analogy with the investment multiplier $\frac{1}{1-\alpha}$, concerning the way in which velocity behaves. It follows from the particular dynamic series which underlies this expression (i.e., $1, \gamma, \gamma^2, \gamma^3, \ldots$ where γ is smaller than unity) that the result of a single new injection of money is that income rises very sharply at the moment of the injection but subsequently falls back to its original level, and that the velocity per income period of the new money falls from one to zero.[2] Correspondingly, continual injections of new money at a constant rate, leading to an ever growing volume of money outstanding, would finally serve merely to finance a constant level of money income per unit period. Thus the average velocity of the money stock outstanding would fall continually even when the "propensity to hoard" was constant for all levels of income, and would fall still faster if the marginal "propensity to hoard" rose as income rose.

This reasoning led Samuelson to conclude[3] that the results to be drawn from multiplier analysis showed an inconsistency between the hypothesis (not necessarily irrefutable) of a stable consumption function[4] used in that analysis, and the hypothesis used by the older writers on monetary theory of a stable velocity; and he seemed to think that this alleged inconsistency was a sign that multiplier and velocity analysis were irreconcilable. Similarly, Goodwin wrote[5] some years

[1] Cf. for example: Machlup, *loc. cit.*; Ackley, *loc. cit.*; Ralph Turvey, "The Multiplier", *Economica*, 1948; Richard A. Goodwin, "The Multiplier Matrix", *Economic Journal*, 1949, esp. pp. 553ff; G. L. S. Shackle, "Twenty Years On: A Survey of the Theory of the Multiplier", *Economic Journal*, 1951, esp. pp. 255-6.

[2] Cf., for example, Samuelson, "Fiscal Policy and Income Determination", *loc. cit.*, p. 593.

[3] *Ibid.*, p. 602.

[4] Although stability of the consumption function has usually been represented as a necessary and sufficient condition for constancy throughout the multiplier series of the coefficient α, it would, as Angell has pointed out (*op. cit.*, p. 193), be more correct to say that a stricter condition is required, namely that the function must be not only stable but also so shaped that, at least over the relevant range of income levels, the marginal propensity to consume is constant—rather than falling as income increases, as Keynes and many others have assumed.

[5] Cf. "The Multiplier", *loc. cit.*, pp. 488-9.

later that: " The multiplier and the quantity hypothesis " (i.e., the hypothesis of the constancy of the velocity of money) " are contradictory. One may describe the multiplier process in velocity terms, but it may, and ordinarily will, require a variable velocity of all money. Consequently velocity has no explanatory value, since it is to multiplier, not velocity, theory that one has to turn for the explanation of the variations ".

It is the purpose of the next section to show that the hypothesis of a stable velocity can be traced to a multiplier sequence which has just as much explanatory value as any other multiplier sequence, and that there is no necessary conflict between this hypothesis and that of a stable consumption function.

IV

THE MULTIPLIER SEQUENCE UNDERLYING THE HYPOTHESIS OF A STABLE VELOCITY

The argument that multiplier analysis shows that velocity cannot be stable at a positive level depends on the supposition that the shape of the hoarding function is exactly analogous to that attributed to the saving function, i.e., that new hoarding occurs in each period and that it bears a certain proportion to the income received in the previous period. Obviously, however, it is in no way necessary, so long as new investment is taking place (and this is what the continual flow of investment injections commonly postulated in multiplier analysis means), that stability of the consumption function, and of its complement the saving function, should imply a hoarding function of the same form as the latter. And evidently the " velocity theorists " were not assuming that it did have this form. What shape did they suppose it to have? Roughly speaking we may say that they assumed that, so long as there was stability of anticipations about future prices, and of current and expected future levels of interest rates, the function relating hoarding to income would be such that the *total hoards outstanding*, but not additions to hoards in each income period, would bear a fairly constant ratio to income. This is a hypothesis which is strongly suggested by, even if it does not expressly underly, the " Cambridge " real balance equation, has occasionally been quite clearly used by one of the exponents of the latter, Sir Dennis Robertson,[1]

[1] We hesitate to say that those who used this equation definitely adopted the hypothesis in question, because although they expressed the " total real balances " which people desired to keep as a proportion of their *given* real income level (and, with a constant income period, this would imply also a certain desired proportion of what we have called " hoards " to that income), they rarely indicated how they expected real balances to move in response to *changes* in the real income level.

Robertson did, however, on at least one occasion (*Banking Policy and the Price Level*, §§ 3 and 4 of the Appendix to Chapter V) explicitly use the hypothesis in question. In treating adjustments of the money supply to increased output, due either to increased productivity per head or to a growth in employment (population), he and has, in addition, been systematically adopted by Professor Angell.[1]

Under this assumption additions to hoards will take place only so long as income continues to increase, and will give way to subtractions (dishoarding) whenever income decreases.[2] Net new hoarding (and dishoarding) thus becomes zero as soon as income settles down to a constant level.

The nature of the multiplier series which underlies velocity, given this assumption about the shape of the hoarding function, may be set out in the following manner:—

Let h represent the ratio of new hoarding (or dishoarding) in any period t to the increment (or decrement) in income $(y(t-1) - y(t-2))$ registered in the period $t-1$. Then, in response to the injection of a single unit of new money, we obtain as our income or multiplier series: $a_1, a_2, a_3, a_4, \ldots a_n$, where

$$\begin{aligned} a_1 &= 1 \\ a_2 &= a_1 - h(a_1 - a_0) \qquad [a_0 = O] \\ a_3 &= a_2 - h(a_2 - a_1) \\ a_4 &= a_3 - h(a_3 - a_2) \\ &\cdot \quad \cdot \quad \cdot \\ &\cdot \quad \cdot \quad \cdot \\ a_n &= a_{n-1} - h(a_{n-1} - a_{n-2}). \end{aligned}$$

When all of the members are reduced to expressions that are multiples of $a_1 = 1$, the series becomes:—

$$1, 1-h, 1-h+h^2, 1-h+h^2-h^3, \ldots, 1-h+h^2-h^3+\ldots \pm h^{n-1},$$

the last term (h^{n-1}) in the nth member of the series being positive when n is an odd number, and negative when n is an even number. Any nth member in the income series is itself the sum of n terms. Summing the positive and negative terms separately and subtracting one sum from the other, we find that it is equivalent to:—

$$S_n = S_{2m} = \frac{1-(h^2)^m}{1-h^2} - \frac{h(1-(h^2)^m)}{1-h^2} = \frac{1-h^n}{1+h} \quad \text{when } n \text{ is even,}$$

$$\text{or } S_n = S_{2m+1} = \frac{1-(h^2)^{m+1}}{1-h^2} - \frac{h(1-(h^2)^m)}{1-h^2} = \frac{1+h^n}{1+h} \text{ when } n \text{ is odd.}$$

speaks of the public presumably wanting to add to their absolute real hoards an amount sufficient to maintain the old proportion to real income.

We should in any case keep in mind that all of the economists concerned would want to add that the hypothesis of a constant ratio of hoards to income is only a very rough approximation to the truth, since the size of hoards is undoubtedly influenced by other factors besides the level of income, e.g. the level of other assets, the level of interest rates, " involuntary " accumulations of hoards in certain circumstances, as well as changes in expectations about future prices.

[1] Thus Angell supposes (*op. cit.*, p. 161) that " when anticipations are constant " hoards will be a constant proportion of total assets, and argues that movements of income may be taken as a rough index of movements in assets, and that consequently we may infer that the desired hoards will under stable conditions tend to be a constant fraction of income.

[2] As Angell has pointed out (*op. cit.*, pp. 175ff), the dynamic series underlying the multiplier $\frac{1}{1-\gamma}$ could result only from continual upward shifts in the hoarding function when the latter is of the kind here assumed.

In either case $S_n \longrightarrow \frac{1}{1+h}$ as $n \longrightarrow \infty$ provided $h < 1$.

The series representing the *new* hoarding (+) or dishoarding (–) in the successive periods is the following:—

$O,\ h(a_1 - a_0),\ h(a_2 - a_1),\ h(a_3 - a_2), \ldots\ldots\ldots\ldots,\ h(a_{n-1} - a_{n-2})$.

Again reducing the terms to multiples of $a_1 = 1$, we obtain the series: $O, + h, - h^2, + h^3, \ldots$, the nth term being $+ h^{2m-1}$ when $n = 2m$, and $- h^{2m}$ when $n = 2m + 1$. The nth term tends to zero as n tends to infinity. The sum of n terms in this series represents the total (new) hoarding *outstanding* in the nth period. This tends to $\frac{h}{1+h}$ as n tends to infinity.

It will be noticed that the income series is an oscillating series,[1] and that, provided the hoarding coefficient (h) is less than unity, the oscillations become continually smaller as time goes on. The *untruncated* multiplier (or velocity), representing the total income which the unit of money creates over an infinite number of unit periods, obviously tends to infinity. The truncated multiplier (or velocity) referring to the total income created over the first n unit periods, where n is any finite number, increases continually as n increases but is, of course, always finite. The truncated multiplier (or velocity) relating to a *single unit period* oscillates between 1 and $1-h$ and finally settles down permanently at around $\frac{1}{1+h}$. Theoretically this " final " value is never reached: using a procedure similar to that applied in investment multiplier analysis, we can, however, calculate the number of unit periods, n, after which the velocity for the single unit period will always fall within limits that are smaller than 10 per cent. above or below the " final " value.[2] The number of unit periods required to reach this approximation to the " final " velocity is the lower the smaller is the hoarding coefficient or, that is, the higher is the " final " velocity.

We thus obtain the result which " velocity theorists " gave us to expect, namely that only a certain proportion $\left(\frac{h}{1+h}\right)$ of the new money finally goes into hoards, and that the remainder $\left(\frac{1}{1+h}\right)$ goes into circulating once each income period in the formation of income. Thus even a single injection of new money provides a permanent

[1] The oscillations will, of course, be reduced if the hoarding outstanding in any period is determined not as a fixed proportion of the income of a single period but as a fixed proportion of the income of a series of periods.

[2] The formula is :

$$n = \frac{\log .1}{\log h}$$

obtained by setting $\frac{1+h^n}{1+h} = 1.1 \left(\frac{1}{1+h}\right)$ or $\frac{1-h^n}{1+h} = .9 \left(\frac{1}{1+h}\right)$

increase in income per unit period. The multiplier itself, that is to say, here acts as a force tending gradually to stabilise the income level.

The effect on income of continual injections of a constant magnitude may be determined by the same summation procedure as is used in all multiplier analysis. So long as new money continues to be created, the income per unit period will under our present assumptions increase without limit. A simple calculation shows also that the average velocity of the growing stock of new money outstanding will fall period by period (but without oscillating) from 1 to the limit $\frac{1}{1+h}$. Thus it is here the average velocity, and not the income level, which settles down to a stable figure in response to constant new injections of money.

We may see the characteristics of the series more clearly with the aid of an arithmetical example. Let us suppose that $h = \frac{1}{2}$. The income series given by the single injection is then:—

$$1, \tfrac{1}{2}, \tfrac{3}{4}, \tfrac{5}{8}, \tfrac{11}{16}, \tfrac{21}{32}, \ldots$$

and the nth term tends to $\frac{2}{3}$ as n tends to infinity.[1] The income (or velocity) per unit period thus oscillates between a maximum of 1 and a minimum of $\frac{1}{2}$, and finally settles down to approximately $\frac{2}{3}$. Theoretically it will take an infinite time for the velocity per unit period of the new money to reach this " final " value. By substituting $h = \frac{1}{2}$ in the formula given above,[2] however, we find that by the time four income periods have passed following the creation of the new money, its velocity per unit period will already be within 10 per cent. above or below the " final " figure.

We conclude, then, that under the assumption here made about the hoarding function our " new money multiplier ", or marginal velocity, tends to $\frac{1}{1+h}$ per income period (where $h < 1$) or to $\frac{N}{1+h}$ per year, where N represents the number of income periods in one year. And so long as the marginal velocity is the same as the average,[3] the velocity of the new money will tend to the same level as that of the old, and we have as our fourth " static " multiplier equation:[4]

$$Y = M \frac{1}{1+h}$$

[1] We notice the strong contrast between this series and the series $1, \gamma, \gamma^2, \gamma^3, \gamma^4, \ldots$ where $\gamma = \frac{1}{2}$, which is:
$1, \frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{16}, \ldots$
and in which the nth term (representing the velocity per unit period of the new money) tends to zero as n tends to infinity.

[2] Cf. footnote 2 on p. 36.

[3] There is a certain amount of evidence in the literature that some people have thought that, at least at certain stages of economic development, the ratio of hoards to income will rise as income rises. In that case the marginal velocity will be below the average, and the average velocity of the whole money stock will gradually fall as income grows. The fall will, however, be much slower than that assumed by certain of the multiplier theorists.

[4] Cf. p. 32 above.

where Y is the income of one income period, and M the total money stock.

We have so far attached the condition to our multiplier series that the coefficient, h, should be smaller than one. It is easily seen that this is equivalent to saying that the proportion of new money $\left(\frac{h}{1+h}\right)$ that finally goes into hoards must fall short of one half.[1] If the coefficient is not smaller than one we come up against a complication which is common to all multiplier analysis under similar conditions, namely that the income series no longer converges. In the present case the series oscillates perpetually between 1 and 0 when h is equal to 1, and oscillates more and more widely (between positive and negative values) as n increases, when h is greater[2] than 1.

This is not, however, a reason for jumping to the conclusion that high values for h are an inevitable source of instability in the system. They would be such a source (always provided, of course, that the hoarding function did not shift despite violent ups and downs in the income level) were the banking system obliged to adopt one or other of the two injection patterns (i.e., either the single injection or the stream of injections of constant magnitude) with which multiplier theorists have, for the sake of simplicity, most frequently worked. The fact, however, that the second term in our income series is equal to $1 - h$ immediately suggests that it is possible to stop the oscillations entirely (and this is true for *all* positive values of h including those below one) by the simple procedure of injecting, in the second period, an amount of money (M_2) equivalent to h times the injection (M_1) of the first period, and making no further injections thereafter. This second injection will suffice to stabilise the average velocity of the total new money ($M_1 + M_2$) from the second period on, and to keep the income constant at the new (higher) level achieved in the first period.

But this is the solution to which Robertson introduced us[3] long ago in answering a rather different, but for some purposes much more relevant, question from the one we have been asking, namely, not what will happen if money is injected either quite irregularly or else at a

[1] It may be noted that Angell's admittedly very rough annual estimates of the ratio of hoards to the total stock of money in the United States indicate that, over the period (1899-1939) which he investigated, the ratio always came well below this figure. The maximum he arrives at for any year is approximately 40 per cent. (for 1939). Cf. *op. cit.*, p. 144.

[2] In the extreme case where the demand for hoards of the people into whose hands the new money first comes (say by way of Central Bank purchases of securities) is infinitely elastic with respect to income (i.e. the marginal h is infinite) the velocity of the new money will be zero from the first period on, and there will be no income creation whatever even in the first " round ". There are of course no oscillations in this case.

[3] *Banking Policy and the Price Level* (1926), §3 of the Appendix to Chapter V. We need of course to allow for differences in terminology. Thus Robertson's " hoards " cover all cash balances (including, that is, both those that turn over once per income period in the creation of income and those which are " hoarded " in our more restricted sense of the term) ; his " day " takes the place of our income

perfectly constant rate, but at what rate *must* money be injected in order to keep the flow of money income permanently at a new higher level corresponding to a given increase in the rate of output?

The solution can easily be generalised to take care of repeated increases in the income level. In order to finance an income level which is increasing at a constant (arithmetic) rate of, say, one per unit period, it will be necessary to inject one unit of money in the first period followed by $1 + h$ in each successive period. The average velocity per unit of the growing stock of new money outstanding again falls gradually (without oscillating) from 1 in the first period (when income increases for the first time) to the limit $\frac{1}{1+h}$.[1] Velocity can, however, as before be *immediately* stabilised at the latter value from any given period onwards by reducing the injection in that period to h, and making no further injections in subsequent periods. It will be noticed that the money injections series here involved diverges very little from a constant series: only the first and last terms are irregular, and they are so only because of the one-period lag which we have postulated between the change in income and the adjustment of hoards. In the absence of such a lag velocity would always be invariable under the assumption we have made about the shape of the hoarding function.

The final question to which the argument of this section leads up is what our assumption about the shape of the hoarding function, and its corollary that in all practically important cases (aside that is from those where it leads to undamped oscillations) velocity tends to stabilise itself at a positive level, implies with respect to possible shapes of the consumption function. Our assumption means that the part of income which is not hoarded (i.e., is available for the remaining purposes of spending on consumption and investment and, possibly, repaying bank debts)[2] is a constant proportion of income (100 per cent.) only

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period, and his " circulation period " is equivalent to $1+h$ times our income period. We have throughout assumed that people adjust their hoards fully to the desired proportion of income in one income period or " day ", i.e. in the period immediately following that in which the change in income first occurs. Robertson's formula is slightly more general, allowing as it does for the possibility that people may spread the process of adjustment over a number of " days ", in which case the injections must be similarly spread.

[1] It would evidently require a constantly rising rate of injection to keep velocity permanently above the " normal " level.

[2] Although it falls outside the declared scope of our article, we may call attention to one point which is connected with contractionist processes but which rounds off our discussion of the factors which led many " velocity theorists " to the view that there is a strong tendency for the velocity of money to be a stabler element than the volume of money itself. It is a familiar argument that in periods of contraction hoards tend to become in part a residual element, i.e. that they are accumulated " involuntarily " with a consequent *diminution in the average velocity of the money* stock. In fact the possibility which many holders of surplus cash have of using it to repay bank debts, and thus to destroy part of the volume of money, very much weakens, even if it does not eliminate, this tendency towards involuntary hoarding. So long as this outlet for surplus cash exists, it will be a factor allowing the desired " normal " hoarding ratio to be maintained, and therefore helping to keep velocity stable.

when income is constant: the proportion falls when income rises and rises when income falls. If, however, we keep to the postulate usually made by investment multiplier theorists that the dynamic propensity to consume falls short of unity, and if we ignore cases where our hoarding function gives rise to undamped oscillations, there is no obvious conflict between this assumption and the hypothesis of a stable consumption function when the consumption-income ratio is supposed constant for all income levels, and still less when that ratio is supposed to fall as income rises.

V

A CASE FOR THE MONEY MULTIPLIER EQUATION

The unpopularity which the velocity, or money multiplier, formula has enjoyed in recent years, and the almost exclusive concentration on the investment multiplier, may make it worth while briefly reconsidering some of the factors affecting the relative usefulness of these and other forms of the multiplier equation as analytical tools for dealing with the effects of changes in the level of spending on the flow of money income.

Formally, of course, all of the equations must give us identical results, provided they are all equally well able to take care, under the multiplicand and the multiplier combined, of all the factors influencing the flow of income. The reasons which may cause us to prefer one version to another will evidently depend on what we consider to be the most important conditions that the division between the multiplier and multiplicand should fulfil. These may perhaps be summarised under three points, all of which have been made at some time or other in the literature. First, we should try to preserve the original basis of distinction, which was one between expenditures that are in some way dependent on the level of income (or " derived ") and those that are not thus dependent (i.e., are " autonomous "). This is important if only for the reason that the autonomous category presumably constitutes the controllable, or more easily controllable, element from the policy point of view. Secondly, and for similar reasons, the division ought to be one which defines the " autonomous " expenditures, or injections, in a way which makes it possible to determine their movements statistically. Thirdly, since it is easier, while keeping within the domain of relatively simple formulæ, to handle a fluctuating multiplicand than a fluctuating multiplier coefficient, it will be convenient if the division is also one which places the more stable element under the multiplier and the less stable element under the multiplicand.

The explanation of how the investment multiplier equation came to take precedence over other possible forms is partly historical. The emphasis placed on public works at the time of the earliest discussions of multiplier theory (as for example in Professor Kahn's original article) was a powerful factor causing the accent originally to fall on the type of injection represented by government expenditures. One of the first questions to be raised, however, was what should be done with decreases in private investment that might be caused by the government investment, or, contrariwise, with increases in private investment that might be induced (whether through the acceleration principle or otherwise) by the additional consumption expenditures out of the income created by the government spending. And it was rather generally conceded that these might be treated respectively as reductions in, or additions to, the multiplicand, even though this procedure obviously blurred the distinction between autonomous and derived expenditures, and confused part of the " effect " with the " cause ".[1] It thus gradually became a widely accepted practice to identify the multiplicand or " injections " with " investments in general ". And Keynes justified the granting of the primary rôle to investment on the ground that " it is usual in a complex system to regard as the *causa causans* that factor which is most prone to sudden and wide fluctuations ".[2] The justification seemed the stronger in that this solution left the multiplier rôle to the propensity to consume, which many believed to be fairly stable over relatively long periods. Thus many economists were satisfied that this division fulfilled the second and third of our conditions even if not the first.

A slight remaining difficulty concerned the treatment of government deficit spending which took the form of relief expenditures; and the inclusion of these in the multiplicand led Sir Dennis Robertson[3] to remark that this type of consumption expenditures was appointed " honorary investment ". All " autonomous " consumption expenditures, however they might be defined, could of course be treated by the consumption multiplier equation, but this equation suffered from the same defects as the investment equation in more marked degree, while lacking certain of its virtues.

The feeling which remained with many multiplier theorists, however, of the practical importance of preserving the distinction between autonomous and derived expenditures, has led to various attempts to effect some sort of combination between the investment and consumption multiplier forms. Goodwin, for example, wants to include in the multiplicand all " injections ", including those that are for consumption purposes,[4] while apparently retaining the assumption that all investment expenditures are " injections " and none of them

[1] Machlup is among those who have most stoutly objected to this practice. Cf. his *International Trade and the National Income Multiplier* (Philadelphia, 1943), p. 10n.

[2] Cf. J. M. Keynes, " The General Theory ", *The New Economics*, p. 191.

[3] Cf. D. H. Robertson, " Mr. Clark and the Foreign Trade Multiplier ", *Economic Journal*, 1939.

[4] Cf. " The Multiplier ", *loc. cit.*, p. 482, where he says : " Government and private investment are the most important injections, but some part of war or relief expenditures, as well as consumer spending for durable goods, must be included ".

spending out of income, the latter *evidently consisting entirely* of "ordinary consumer expenditures on perishable goods and services ".[1] Others have sought, rather more systematically, to combine "autonomous investment" with "autonomous consumption" under the multiplicand, and "induced investment" with "induced consumption" under the multiplier. The static multiplier equation corresponding to this division has usually been written in the form

$$Y = A \frac{1}{1 - (\alpha + \beta)}$$

where A represents the "autonomous expenditures", $\alpha + \beta$ the sum of the "propensity to consume" and the "propensity to invest", and $1 - (\alpha + \beta)$ the "propensity to save minus the propensity to invest".[2] Some of the writers using this formula[3] have identified $\alpha + \beta$ with the "propensity to spend" out of income, which we previously called γ, and $1 - (\alpha + \beta)$ with the "propensity to hoard" $(1 - \gamma)$. Quite apart from our objections to the implied shape of the hoarding function, this is, however, incorrect *so long as we do not assume that injections of autonomous expenditures are synonymous with injections of new money.*[4]

Two inconveniences stood in the way of the division we have just described if it was really to be one which defined autonomous expenditures as something different from expenditures out of new money. One of these was the obvious difficulty of finding a criterion for separating the autonomous from the derived expenditures—a criterion that would allow us to recognise and measure changes in them. One distinction which early suggested itself was that between government deficit spending on the one hand and all other forms of spending on the other; but this was obviously only of interest in certain historical situations. A more recently developed classification, applying to investment expenditures, defines autonomous investment as springing "notably from changes in technique" and induced investment as

[1] *Ibid.*

[2] Cf. Samuelson, "Fiscal Policy and Income Determination", *loc. cit.*, pp. 577f; and Lange, *loc. cit.*, pp. 231ff.

[3] As for example Lange (*ibid.*).

[4] For so long as we assume that autonomous expenditures are not necessarily coincident with spending out of new money we must allow that the excess of saving over induced investment, represented by the expression $1-(\alpha+\beta)$, may go in whole or in part into financing the autonomous investment or consumption expenditures (under A) rather than into hoards. And conversely, we must allow that part of the investment referred to under β (and perhaps part of the consumption under α) though "induced" by the level of income may not be paid for "out of" income but may be financed instead by new money. All that this means is that, so long as we distinguish the two equations, part of the expenditures that appear under the multiplicand in the money multiplier equation appear under the multiplier in the autonomous spending multiplier equation, and *vice versa.* It also means that the *dynamic* multiplier coefficient $\alpha+\beta$ may be greater than unity, a condition that has usually been excluded by multiplier theorists, probably by reason of a mistaken belief that the fact that the *static* multiplier coefficient cannot assume such values implies a similar limitation on the dynamic coefficient.

" the result of an increase in final demand or sales volume ":[1] this is, however, a criterion to which, whatever its analytical validity, it is difficult to give statistical content.

The remaining difficulty was the hesitation which many people felt in accepting the proposition, which was necessary in order to give a stable multiplier coefficient, $\alpha + \beta$, that the " propensity to invest ", as well as the " propensity to consume ", might be treated as a constant.

It seems fair to conclude that the search for a multiplier formula which is of general application, and which meets all the three tests of a satisfactory division between the multiplier and the multiplicand, leads us back almost inevitably to the money multiplier formula. Two reasons speak strongly in favour of this solution. In the first place, the clearest division of total expenditures into " autonomous " and " derived " which we can hope to make is probably still that which identifies the first with those that are due to the injection of new money into the system, and the second with those that are due to spending out of income, or out of the existing stock of money.[2] In the second place, many will feel, failing empirical proofs to the contrary, that there is still some justification for assuming that the money multiplier coefficient is approximately constant, or shows only small variations about a trend, so long as general economic conditions are fairly stable. This assumption follows from a hypothesis concerning the shape of the hoarding function, and its tendency towards stability so long as expectations are constant, which is probably no more difficult to defend than is the hypothesis of a stable consumption function; and it does not necessarily conflict with the latter.

We are thus back at the old velocity formula. " Velocity theorists " would, we should however add, point rather emphatically at the necessity of qualifying the statement that velocity tends to be stable, so as to allow for effects of shifts, either spontaneous or induced, in the hoarding function. In fact a considerable part of their analysis has been concerned with the factors that are likely to cause marked shifts even over relatively short periods, and among these factors they

[1] Alvin H. Hansen, *Business Cycles and National Income* (New York, 1951), p. 190. It should be noted, however, that Professor Hansen himself keeps to the practice of putting induced as well as autonomous investment (and government outlays) into the multiplicand, and only induced consumption into the multiplier. Cf. also J. R. Hicks, *The Trade Cycle* (Oxford, 1950), pp. 59ff.

[2] This formula has sometimes been criticised on the grounds that it fails to take account of the importance of the method of injection, i.e. of the fact that it matters into whose hands the new money first comes, as for example whether, in the first stages of a " reflation," it comes into the hands of people who have a high propensity to hoard or into those of groups who have a low propensity to hoard. The problem created by the fact that different groups have different propensities (whether to hoard or to consume), so that the average for the community as a whole alters with changes in income (or assets) distribution, is, however, common to all multiplier analysis which abstracts from such changes. All simple multiplier formulæ which treat the economic system in terms of single aggregates are, that is to say, equally rough instruments, and liable to be misleading if we let the appropriate qualifications slip from our sight.

have stressed price fluctuations associated with excessive movements in the volume of money, i.e., in the multiplicand itself.[1] This theme, however, brings us beyond the scope of this article, which was intentionally confined to considering the multiplier mechanism under the supposition that movements in the multiplicand (upwards) were kept short of the boundary where " unemployment economics " ends and inflation begins, and contained no place at all for deflationary movements. None the less, it is pertinent to remark that many of the factors which " velocity theorists " have considered as likely to shift the hoarding function are among those which we might also expect to cause shifts in the consumption function.

We conclude, then, that while it may very well be useful for certain purposes and in certain conditions to separate out one class of expenditures (such as investment, or some particular type of investment) as the prime mover of changes in the flow of money income, each of the formulæ which does this is only of limited applicability, and that in many cases it will be more useful to take the formula in which the quantity of money is the multiplicand and the income velocity the multiplier. It has been the purpose of the present article to show that this last formula (to which many of those who explicitly claim to use the investment multiplier implicitly have recourse) is just another species of the multiplier genus, and not a different type of analysis with less explanatory value. This is not to deny that here too there are many practical and conceptual difficulties in drawing hard and fast demarcation lines between the multiplicand, the multiplier, and the multiplier period, e.g., of defining unequivocally the volume of money as distinguished from the velocity of that money, or of determining what changes in the latter are due to changes in the income period and what due to changes in the propensity to hoard. But in this it is not different from the investment multiplier, as the perennial discussion of the corresponding problems in the literature shows.

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[1] They have also observed that the income period might undergo marked changes under such conditions, thus affecting the ratio of velocity per calendar period to velocity per income period.

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[Footnotes]

[4] **The Multiplier Time Period: Money, Inventories, and Flexibility**
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